ACTIVE CONTROL OF SUBHARMONIC TRANSITION OF BOUNDARY LAYER

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Boundary-layer transition to turbulence arises as a result of the growth of sinusoidal disturbances. Two types of transitions are known at present: K-type occurs through the formation and the destruction of strong waves and is characterized by the excitation of the high-frequency component of the spectrum; the C-type occurs at much lower level of initial disturbances and is accompanied by the growth in low-frequency spatial fluctuations [1]. The behavior of the boundary layer in this case could be explained by resonant interaction of disturbance waves [2-6]. Approbation of the models requires further validation with experimental results.

The study of transition mechanism is important to develop methods for its control. Usually such a control is effected by changing the mean-velocity profile and the boundary conditions of the flow. In recent years active control using the effect of wave interference [7-9] has gained importance. The spectral component is detected at the initial stage of transition and its strength is regulated by the downstream introduction of waves with specific amplitude-phase characteristics.

The effectiveness of the method is obviously dependent on the type of transition and the conditions for the excitation of control signal. It is possible to expect that the maximum effect is realizable with C-type transition, where the quasilinear development of the particular wave is present [3, 4].

Such conditions were realized, in particular, in the experiment [7]. Two vibrating ribbons were located at different distances from the leading edge of a flat plate boundary layer. One of them, I, simulated natural disturbance causing transition and the second, II, located downstream, played the role of control. The disturbance spectrum recorded the growth of subharmonics, characteristic of C-type transition.

The present paper uses the resonant interaction of Tollmien–Schlichting (TS) waves as the basis of the transition mechanism to compare with experimental data [7] and conditions for effectiveness of interferential control technique are analyzed. The guiding mechanism of the C-type transition is the resonant interaction of plane TS-waves with spatial waves in symmetric triads. An isolated triad is an elementary model that permits the description of the behavior of the principal components of the disturbance spectrum [3, 4].

The velocity field in control experiments (CE) may be expressed by nondimensional streamfunction:

$$\Psi = \Psi_0 + \varepsilon \hat{B} e^{\gamma_1 t} \varphi_1 (y, \mathrm{Re})\, e^{i\theta_1} + \varepsilon \hat{A} e^{\gamma t} \varphi (y, \mathrm{Re})\, (e^{i\theta_+} + e^{i\theta_-}) + o(\varepsilon^2), \tag{1}$$

where $\varepsilon \ll 1$; $\theta_1 = \int \alpha_1 dx - \omega_1 t$; $\theta_\pm = \int \alpha\, dx - \frac{\omega_1}{2} t \pm \beta z$; Re is the Reynolds number based on displacement thickness; $\Psi_0(y, \mathrm{Re})$ is the Blasins profile; $\varphi_j(y, \mathrm{Re})$ and the relation $\Omega_j(\alpha, \beta, \mathrm{Re}) = \omega_j + i\gamma_j$ are determined by the solution of Orr–Sommerfeld equations; complex amplitudes $B = \hat{B} \exp \gamma_1 t$, $A = \hat{A} \exp \{i \int (\alpha - \alpha_1/2) dx + \gamma t\}$ satisfy the system [4]

$$\left(v_1 \frac{\partial}{\partial x} - \gamma_1\right) B = S_1 A^2, \quad \left(v \frac{\partial}{\partial x} - \gamma - iv(\alpha - \alpha_1/2)\right) A = SBA^*, \tag{2}$$
$$A(x_0) = A_0,\ B(x_0) = B_0,\ A(x_1 - 0) = A_{10},\ B(x_1 - 0) = B_{10},$$
$$A_1 \equiv A(x_1) = A_{10} + A_2,\ B_1 \equiv B(x_1) = B_{10} + B_2,$$
$$\arg B_2 - \arg B_{10} = \Delta,\ \arg A_2 - \arg A_{10} = \Delta_1,\ |A| = a,\ |B| = b,$$

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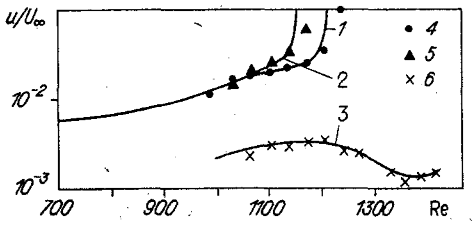

Fig. 1

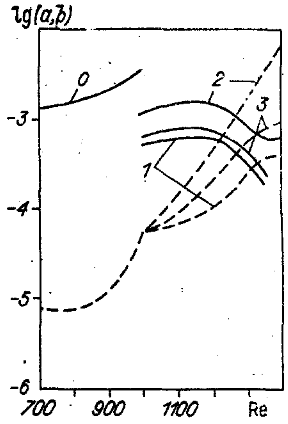

Fig. 2

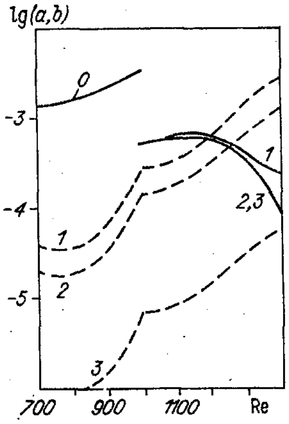

Fig. 3

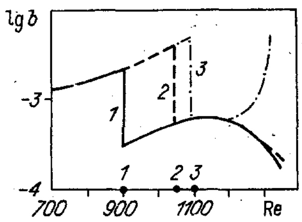

Fig. 4

x_0 and x_1 correspond to the section where the vibrators I and II are located in CE. Here, it is assumed that the second vibrator does not change the spectrum and the spatial structure of wave disturbances. The control mechanism results in a change in their amplitudes.

A direct comparison with test results [7] is shown in Fig. 1. Curves 1-3 denote streamwise velocity component $u = b\tilde{u}(x)$ [$\tilde{u} = \max u(x, y)$ along y] when the ribbons operate independently and the result when they operate together, respectively; 4-6 indicate experimental data from [7].

A characteristic feature observed under the independent conditions is the sharp increase in growths. Furthermore, the generator I located upstream of II leads to such a growth at larger values of Re. In [7] it was noted that it was not possible to completely "destroy" waves in CE.

These computed characteristics may be explained within the framework of the chosen model. The behavior of amplitudes under autonomous conditions is typical for resonant interactions observed in C-type transition [1, 2, 4]. The dominating disturbance growth from the generator II can be explained by higher initial level of subharmonics at $x = x_1$. (Unlike the section x_0, in whose neighborhood such waves damp out rapidly, the neighborhood of x_1 makes the region unstable. This reduces their excitation and extends the spectrum of α which is equivalent to the growth of initial amplitudes [4].) Computed curves 1 and 2 illustrate close agreement with the experiment at $a_0 \approx 5 \cdot 10^{-6}$ and $a_1(x_1) \approx 2 \cdot 10^{-4}$. The value of $\theta = \tan^{-1}(\beta/\alpha) \approx 50°$ was obtained from the condition for maximum effective resonance in triad; initial amplitudes of subharmonics were determined from amplitudes and amplification rates for two-dimensional waves, known from experiment [7]. These parameters were retained in the computation of the conditions for CE. The corresponding curve 3 practically coincides with measurements when $a_0 = 5 \cdot 10^{-6}$, $a_1 = 1.8 \cdot 10^{-4}$, $b_0 = 6 \cdot 10^{-3}$, and $b_1 = 2 \cdot 10^{-3}$. The frequency parameter corresponds to the experimental value $F = 110 \cdot 10^{-6}$. The total effect of the combined operation of the two generators depends on the degree of coherence of excited waves. Its absence due to nonuniformities and the inertia of the ribbons is considered the principal cause of "nondestructibility" of the introduced perturbations and the impossibility of correctly specifying $b_{10} = b_2$ and $\Delta = \pi$. In computations, $b_{10} = b_2$ and $\Delta = 0.96\pi$. A decrease in $a_1(x_1)$ also appears to be the result of $\Delta_1 \approx \pi$. The characteristic amplification of the amplitude (curve 3) when $Re \geq 1350$ is explained by a jump in the two-dimensional wave in the field of strong spatial subharmonics.

Let us return to the analysis of the effectiveness of the control technique as a function of the parameters of vibrators and the background noise level. The control vibrator II is assumed to be ideal ($A_2 = 0$) and $\theta = 50°$, $F = 11 \cdot 10^{-5}$ are fixed (variations in θ and F over a wide range have little effect on the nature of evolution of perturbations [3]).

The degree of suppression of disturbances also significantly depends on small disagreements $b_{10} - b_2 \neq 0$, $\Delta \neq n\pi$, $n = \pm 1, \pm 3, \ldots.$ Figure 2 shows the behavior of b(x) (continuous line) and the amplitude of subharmonics $a(x)$ (dashed lines) when $b_2 = b_{10}, 0.96b_{10}$, $\Delta = 0.96\pi$, 0.9π, and 0.96π (lines 1-3). Curve 0 in Figs. 2 and 3 denotes initial two-dimensional wave.

The parametric condition for the amplification of spatial disturbances in the linear wave field is realized under the above conditions in the region $x_0 \le x \le x_1$. The control effect at $x = x_1$ results in a decrease in its amplitude. As a result, the growth of subharmonics is reduced, and the evolution of excited wave remains practically independent. The transition process is extended.

The picture changes if in the region $x \le x_1$ the wave amplitudes manage to equalize. Nonlinear effects become significant with an explosive growth of all perturbations [3, 4]. The drop in the amplitude of plane waves (at x_1) does not stabilize the disturbance growth. Obviously, active control is possible only at the stage preceding the nonlinear growth, and it is associated with the location of the point x_1 and the value of a_0.

The dependence on the initial level of low-frequency disturbances at fixed $x_1 - x_0$, b_0, $b_{10} = b_2$, $\Delta = 0.96\pi$, $A_2 = 0$ is shown in Fig. 3. Subharmonics (dashed lines 1-3) grow rapidly. Nonlinear deviation of the basic wave (continuous line 1) is observed in the interaction with the subharmonic 1, which is of the same order at $x_0 \le x \le x_1$. Lower values of a_0 (dashed lines 2 and 3) practically do not affect the increment B(x) (continuous lines 2 and 3). Note that the parametric amplification of low-frequency region of the spectrum is not discontinued but is appreciably lowered.

For the given types of vibrators and background noise at x_0 it is possible to define $L = x_1 - x_0$ above which suppression is not effective. Figure 4 shows the dynamics of the strength of excited frequency for different locations of the vibrator II [the number of the curve corresponds to the number of the point of location of the vibrator on the axis RE, $a(x_0) = 10^{-5}$, $b_0 = 1.3 \cdot 10^{-3}$, $\Delta = 0.96\pi$]. It is seen that the displacement of the source II for $Re \le 1050$ significantly extends the transition process. The location of this point determines the value of L. It is possible to obtain an estimate of L clearly from the form of A(x), B(x) in the region of parametric growth of the triad [10]:

$$\left|\frac{a(x_1)}{b(x_1)}\right| = \left|\frac{a(x_0)}{b(x_0)} \exp\left[\frac{b(x_0)\, v_1 S}{\gamma v} \exp\int_{x_0}^{x_1} \gamma_1 \frac{dx}{v_1} - \int_{x_0}^{x_1} \gamma_1 \frac{dx}{v_1}\right]\right| \simeq \frac{1}{10}.$$

The determination of L has practical importance for transition control in boundary layer. It is worth noting, however, that these conditions correspond to ideal control-vibrators that do not affect the level of subharmonic disturbances. In realistic conditions this is, apparently, not so. The ribbon excited a spectrum of low-frequency disturbances. These disturbances themselves mutually interact and can cause transition. It is thus possible to recommend the location of the control mechanism in the region of Re where subharmonic perturbations are stable.

The results obtained make it possible to describe the mechanism of CE within the framework of a simple model and confirm the concepts of the resonant-wave nature of the initial stages of C-type transition.

In conclusion, the authors acknowledge the attention given to the work by V. Ya. Levchenko.

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BOUNDARY-LAYER RECEPTIVITY TO ACOUSTIC DISTURBANCES

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It is known that, in the case of small external disturbances, boundary-layer transition from laminar to turbulent flow is caused by the growth of unstable Tollmien—Schlichting (TS) waves [1, 2]. The location of transition region and the nature of transition process essentially depend on boundary-layer receptivity to external disturbances, i.e., on the excitation of TS waves by background noise. Scattering of acoustic waves in spatial flow nonuniformities due to surface roughness or nonuniform boundary conditions (nonuniform wall heating, local mass transfer through porous surface, etc.) is a typical mechanism of unstable wave generation.

Excitation of TS waves by streamwise acoustic wave on a small isolated roughness on a flat plate was experimentally investigated in [3]. Asymptotic analysis of this problem was carried out in [4] for the case when the roughness was located in the neighborhood of the lower branch of the neutral curve. Generation of TS waves by sound on sinusoidal and distributed waviness of the flat-plate surface was considered in [5] at small freestream Mach numbers.

Theoretical investigation of the excitation of TS waves by acoustic disturbance on local three-dimensional roughness in a compressible boundary layer is presented in this present paper. Analysis is carried out by reducing the problem to the solution of a system of eigenfunctions of the linearized Navier—Stokes equations [6, 7]. The generation of unstable waves is the result of weak nonlinear interaction of sound with the flow nonuniformity. Computations on the excitation of TS waves on individual roughness element in the flat-plate boundary layer agree well with experimental data [3].

If isolated roughnesses are small, or if they are far from the point of instability so that the final amplitudes of the generated TS waves are small, then it is possible that the distributed generation of unstable waves may become dominant. In this case the excitation is caused by acoustic scattering on weak nonuniformity due to nonparallel flow conditions in the boundary layer [8-10]. A comparison of the effectiveness of TS-wave generation by sound on isolated roughness and on distributed roughness is given in this paper.

1. Consider two-dimensional compressible flow. A roughness element in the form of a small hump, which is a stationary disturbance source in the boundary layer, is located at a disturbance L from the leading edge of the flat plate. External acoustic wave with

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